PHYC 581: High Energy Astrophysics
Fall 2014

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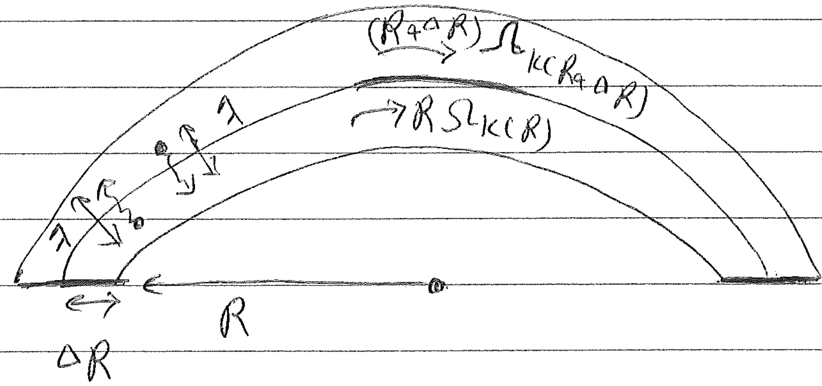
Accretion Disk Theory:

Whether a compact object accretes from the surrounding medium, or from a binary companion, the accreting plasma settles into a disk perpendicular to its net angular momentum vector. Even if the accretion proceeds spherically at first, cooling processes eventually dissipate the plasma's support in the parallel direction away.

Thin Disk Theory:

To describe the structure of the disk, we first find hydrodynamic equations similar to those derived in the case of spherical accretion. We will make full use of the specific geometry in the Keplerian motion.

Consider a geometrically thin disk rotating close to an orbital plane:



At any radius R, the matter rotates in a ring with a circular velocity $v_\phi(R) = R\Omega_k(R)$, where:

$$\Omega_k(R) = \left(\frac{GM}{R^3}\right)^{\frac{1}{2}}$$

This is the Keplerian angular velocity. The mass and angular momentum of the ring at radial distance R are:

Mass $= 2\pi R \Delta R \Sigma$, Angular Momentum $= (2\pi R \Delta R \Sigma) R^2 \Omega_k(R)$

Here $\Sigma = \rho H$ is the surface density, with "H" being the thickness of the disk. The equation for the conservation of mass is:

$$R\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}(R \Sigma v_R) = 0$$

Here v_R is the radial velocity (inward) due to accretion.

The equation for the angular momentum is:

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega)+\frac{\partial}{\partial R}(R\Sigma v_R R^2\Omega)=-\frac{1}{2\pi}\frac{\partial T_{out}}{\partial R}$$

Here T_{out} is the viscous torque that a ring exerts to the neighboring outer ring, which is given by:

$$T_{out}\sim 2\pi R\Sigma\tilde{v}\,R(R+\lambda)\left[\Omega(R)-\Omega(R+\lambda)\right]$$

λ and $\tilde{v}$ are the length scale and speed, respectively, for matter crossing between the neighboring rings. This can be due to thermal motion or turbulence. For example, in case of thermal motion, λ represents the mean-free-path of particles in the gas. By defining the coefficient of kinematic friction $\nu\equiv\lambda\tilde{v}$, we have:

$$T_{out}\simeq-2\pi\nu\Sigma R^3\Omega' \qquad \left({}'\equiv\frac{\partial}{\partial R}\right)$$

After using this expression in the equation for the conservation of the mass, the angular momentum equation can be written as follows;

$$R\Sigma v_R (R^2\Omega)' = \frac{\partial}{\partial R}(R\nu\Sigma R^2\Omega')$$

For a Keplerian flow $\Omega = \Omega_K = \left(\frac{GM}{R^3}\right)^{1/2}$, which results in:

$$v_R = \frac{-3}{\Sigma R^{1/2}} \frac{\partial}{\partial R}(\nu\Sigma R^{1/2})$$

In situations where ν is constant and the disk is steady state, the mass conservation implies that $R\Sigma v_R = $ Const., and hence the accretion rate follows:

$$\dot{M} = -2\pi R\Sigma v_R = \text{Const.}$$

Also:

$$v_R \sim O\left(\frac{\nu}{R}\right) \Rightarrow \frac{v_R}{c_s} \sim \alpha\left(\frac{H}{R}\right)$$

Here we have used the useful parametrization $\nu \equiv \alpha c_s H$.

For thin disks $H \ll R$, hence $v_R \ll c_s$, which validates the assumption of quasi-Keplerian motion of the disk. Thus, although the shear viscosity leads to outward transfer of angular momentum and inflow of mass, the dominant velocity at any given radius is alway azimuthal. As we shall see soon, by analyzing the vertical structure of thin disks, we typically have $\frac{H}{R} \lesssim O(0.01)$, which implies the disks are very thin.